# USIM

## FINAL EXAMINATION
## SEMESTER II, ACADEMIC SESSION 2018/2019

| DATE : JUNE 2019 | DURATION : 2 HOURS 30 MINUTES |
|---|---|

# SMG2013
## PROBABILITY
## (KEBARANGKALIAN)

**INSTRUCTIONS TO CANDIDATES:**

1. Answer **all** questions in the **answer booklet** provided.
2. All answers must be written in English.
3. All answers must be clearly written and readable.
4. Candidates are **not allowed** to take the question papers out of the examination hall.
5. Please complete your particulars in **Borang H.**

**DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE INSTRUCTED TO DO SO**

This question paper has **four (4)** printed pages excluding this cover page

**[70 MARKS]**

**ANSWER ALL QUESTIONS IN THE ANSWER BOOKLET PROVIDED.**

1. a. An actuary has discovered that policyholders are three times as likely to the file two claims as to file four claims. If the number of claims filed has Poisson distribution, find the variance of the number of claims filed.

(4 Marks)

b. If you roll a pair of fair dice, find the probability that

i. the first 11 occurs on the eighth roll.

(5 Marks)

ii. the third total number less than 7 occurs on the eleventh roll.

(5 Marks)

**[Total: 14 Marks]**

2. A professor gives a test to a large class. The time limit for the test is 50 minutes, and the first student to finish is done in 35 minutes. The professor assumes that the random variable $X$ for the time it takes a student to finish the test is uniformly distributed over [35, 50].

a. State the density function of $X$.

(2 Marks)

b. Find the mean and standard deviation of $X$.

(3 Marks)

c. Estimate the time $X$ that 60 percent of the students will finish their test.

(5 Marks)

**[Total: 10 Marks]**

3. A diagnostic test for the presence of a disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let $X$ denote the disease state of a patient, and let Y denote the outcome of the diagnostic test. The joint probability function of $X$ and $Y$ is given by:

$$P(X=0,Y=0)=0.800,\quad P(X=0,Y=1)=0.025,$$
$$P(X=1,Y=0)=0.050,\quad P(X=1,Y=1)=0.125.$$

Calculate $Var(Y|X=1)$.

**[Total: 10 Marks]**

4. Let $X$ and $Y$ be continuous random variable with joint density function as follows:

$$f(x,y)=\begin{cases}\frac{8}{3}xy & 0\le x\le 1, x\le y\le 2x\\ 0 & \text{otherwise}\end{cases}$$

Find

a. $E(X)$ and $E(Y)$.

(12 Marks)

b. $E(XY)$.

(6 Marks)

c. Covariance of $X$ and $Y$.

(2 Marks)

**[Total: 20 Marks]**

5. a. Let $X$ be a discrete random variable with the following probability distribution function:

| $x$ | $f(x)$ |
|---|---|
| 0 | 0.20 |
| 1 | 0.15 |
| 2 | 0.25 |
| 3 | 0.40 |

Find the moment generating function at $t$ = 3.

(4 Marks)

b. An actuary models the lifetime of a device using the random variable $Y = 10X^{0.8}$ where $X$ is an exponential random variable with mean 1 year. Let $f(y)$ be the density function for $Y$. Determine $f(y)$ for $y > 0$.

(5 Marks)

c. An actuary determines that the claim size for a certain class of accidents is a random variables, $X$, with moment generating function.

$$M_X(t) = \frac{1}{(1-2500t)^4}$$

Determine the standard deviation of the claim size for this class of accidents.

(7 Marks)

**[Total: 16 Marks]**

**END OF QUESTIONS**

APPENDIX - MATHEMATICAL FORMULA

| Distribution | Formula | Mean, $\mu$ | Variance, $\sigma^2$ |
|---|---|---|---|
| Discrete Uniform | $f(x;k)=\frac{1}{k},\quad x=x_1,x_2,\ldots,x_k$ | $\frac{1}{k}\sum_{i=1}^{k}x_i$ | $\frac{1}{k}\sum_{i=1}^{k}(x_i-\mu)^2$ |
| Multinomial | $f(x_1,x_2,\ldots,x_k;p_1,p_2,\ldots,p_k,n)=\binom{n}{x_1,x_2,\ldots,x_k}p_1^{x_1}p_2^{x_2}\ldots p_k^{x_k}$ | - | - |
| Hypergeometric | $h(x;N,n,k)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$ | $\frac{nk}{N}$ | $\frac{N-n}{N-1}.n.\frac{k}{N}\left(1-\frac{k}{N}\right)$ |
| Negative Binomial | $b*(x;k,p)=\binom{x-1}{k-1}p^k q^{x-k}$, for $x=k,k+1,k+2,\ldots..$ | $\frac{k}{p}$ | $\frac{kq}{p^2}$ |
| Geometric | $g(x;p)=pq^{x-1}$, for $x=1,2,3,\ldots..$ | $\frac{1}{p}$ | $\frac{q}{p^2}$ |
| Poisson | $p(x;\lambda)=P(X=x)=\frac{(\lambda)^x e^{-\lambda}}{x!}$, $x=0,1,2,\ldots$ | $\lambda$ | $\lambda$ |
| Continuous Uniform | $f(x;A,B)=\begin{cases}\frac{1}{B-A}, & A\le x\le B\\ 0 & \text{elsewhere}\end{cases}$ | $\frac{A+B}{2}$ | $\frac{(B-A)^2}{12}$ |
| Exponent | $f(x;\beta)=\begin{cases}\frac{1}{\beta}e^{-x/\beta}, & x>0\\ 0 & \text{elsewhere}\end{cases}$, $\beta>0$ | $\beta$ | $\beta^2$ |

Dicetak oleh:

Unit Peperiksaan dan Pengijazahan
Bahagian Pengurusan Akademik
Universiti Sains Islam Malaysia