USIM

FINAL EXAMINATION
SEMESTER I, ACADEMIC SESSION 2019/2020

DATE: JANUARY 2020	DURATION : 2 HOURS 30 MINUTES

SMG2013
PROBABILITY
(KEBARANGKALIAN)

INSTRUCTIONS TO CANDIDATES:

1. Answer **all** questions in the **answer booklet** provided.
2. All answers must be written in English.
3. All answers must be clearly written and readable.
4. Candidates are **not allowed** to take the question papers out of the examination hall.
5. Please complete your particulars in **Borang H.**
6. The mathematical formula and statistical table are provided at the end of this question paper.

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE INSTRUCTED TO DO SO

This question paper has **four (4)** printed pages excluding this cover page

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QUALITY SYSTEM SIRIM CERTIFIED TO ISO 9001:2015 CERT. NO. :QMS 02164

QUALITY SYSTEM SIRIM CERTIFIED IQNet MANAGEMENT SYSTEM CERTIFIED TO ISO 9001:2015 CERT. NO. :QMS 02164

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HAKCIPTA TERPELIHARA USIM

[70 MARKS]

ANSWER ALL QUESTIONS IN THE ANSWER BOOKLET PROVIDED.

1. a. A foreign student club lists as its members 2 Canadians, 3 Japanese, 5 Italians, and 2 Germans. If a committee of 4 is selected at random, find the probability that

   i. all nationalities are represented;

   (3 Marks)

   ii. all nationalities except Italian are represented.

   (6 Marks)

   b. Three people toss a fair coin and the odd one pays for coffee. If the coins all turn up the same, they are tossed again. Find the probability that fewer than 3 tosses are needed.

   (5 Marks)

   [Total: 14 Marks]

2. The daily amount of coffee, in liters, dispensed by a machine located in an airport lobby is a random variable Y having a continuous uniform distribution from 7 to 10.

   a. State the density function of Y.

   (2 Marks)

   b. Find the mean and standard deviation of Y.

   (2 Marks)

   c. Determine the conditional probability of $P(Y > 7.2 | Y \leq 9)$.

   (6 Marks)

   [Total: 10 Marks]

3. A car dealership sells 0, 1, or 2 luxury cars on any day. When selling a car, the dealer also tries to persuade the customer to buy an extended warranty for the car. Let X denotes the number of luxury cars sold in a given day, and let Y denote the number of extended warranties sold.

$$P(X=0,Y=0)=\frac{1}{6};\ P(X=1,Y=0)=\frac{1}{12};\ P(X=1,Y=1)=\frac{1}{6};$$
$$P(X=2,Y=0)=\frac{1}{12};\ P(X=2,Y=1)=\frac{1}{3};\ P(X=2,Y=2)=\frac{1}{6}.$$

Arrange all probabilities in table and show the marginal distribution function of X and Y.Then, find the variance of X.

[Total: 15 Marks]

4. A device runs until either of two components fails, at which point the device stop running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x,y)=\begin{cases}\dfrac{x+y}{8} & ;0<x<2, 0<y<2\\ 0 & \text{elsewhere}\end{cases}$$

Find the probability that the device fails during its first hour of operation.

[Total: 10 Marks]

5. Five balls numbered 1, 2, 3, 4 and 5 are placed in a bag. After the balls are mixed, one is selected, its number noted and then it is replaced. If this experiment is repeated many times, find the kth moment of X where X is the number on the ball. Then, using this kth moment, compute the mean and standard deviation of X.

[Total: 10 Marks]

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6. a. Let the random variable X has the probability density function

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Consider another random variable $Y = X^2$. Find the density function of Y, $f(y)$ for $y > 0$.

(4 Marks)

b. Given the moment generating function of the random variable X having a chi-square distribution with υ degrees of freedom is $M_X(t) = (1-2t)^{-\upsilon/2}$. Using this $M_X(t)$, find the mean and variance of X.

(7 Marks)

[Total: 11 Marks]

END OF QUESTIONS

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APPENDIX - MATHEMATICAL FORMULA

Distribution	Formula	Mean, μ	Variance, σ^2
Discrete Uniform	$f(x;k)=\frac{1}{k}, \quad x=x_1,x_2,\ldots,x_k$	$\frac{1}{k}\sum_{i=1}^{k}x_i$	$\frac{1}{k}\sum_{i=1}^{k}(x_i-\mu)^2$
Multinomial	$f(x_1,x_2,\ldots,x_k;p_1,p_2,\ldots,p_k,n) = \binom{n}{x_1,x_2,\ldots,x_k}p_1^{x_1}p_2^{x_2}\ldots p_k^{x_k}$	-	-
Hypergeometric	$h(x;N,n,k)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$	$\frac{nk}{N}$	$\frac{N-n}{N-1}.n.\frac{k}{N}\left(1-\frac{k}{N}\right)$
Negative Binomial	$b*(x;k,p)=\binom{x-1}{k-1}p^kq^{x-k}$, for $x=k,k+1,k+2,\ldots..$	$\frac{k}{p}$	$\frac{kq}{p^2}$
Geometric	$g(x;p)=pq^{x-1}$, for $x=1,2,3,\ldots..$	$\frac{1}{p}$	$\frac{q}{p^2}$
Poisson	$p(x;\lambda)=P(X=x)=\frac{(\lambda)^x e^{-\lambda}}{x!}$, $x=0,1,2,\ldots$	λ	λ
Continuous Uniform	$f(x;A,B)=\begin{cases}\frac{1}{B-A}, & A\le x\le B\\ 0 & \text{elsewhere}\end{cases}$	$\frac{A+B}{2}$	$\frac{(B-A)^2}{12}$
Exponent	$f(x;\beta)=\begin{cases}\frac{1}{\beta}e^{-x/\beta}, & x>0\\ 0 & \text{elsewhere}\end{cases}$, $\beta>0$	β	β^2

Dicetak oleh:

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